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$$1+3+7+17+38+76+\dots \text{ or } 1+3+7+17+39+81+\dots$$

or as many more as I care to take the time to construct, on the one supposition that it is an arithmetical series of the fourth order. Or let me assume it a recurring series of the fourth order, and get, using the scale of relation,

$$u_n=2u_{n-1}+u_{n-2}+0u_{n-3}+u_{n-4}, \text{ say, } 1+3+7+17+42+104+\dots$$

The fact is, the problem is absolutely without meaning. There is nothing in the sequence $1+3+7+17+\dots$ to show what the next term is. A series is not determined until its law is in some way stated or indicated.

267 Proposed by O. E. GLENN, Ph. D., Philadelphia, Pa.

Express the trigonometric functions of x as infinite continued fractions.

Solution by R. D. CARMICHAEL, Professor of Mathematics, Anniston, Ala.; and J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Let the series $u=A-B+C-D\dots=\cfrac{a_1}{c_1+\cfrac{a_2}{c_2+\cfrac{a_3}{c_3+\dots}}}$

To determine the a's we have the following (Euler's "Introductio," I, §367):

$$a_1=A.c_1,\ a_2=\frac{B.c_1c_2}{A-B},\ a_3=\frac{AC.c_2c_3}{(A-B)(B-C)},\ a_4=\frac{BD.c_3c_4}{(B-C)(C-D)}\dots$$

in which the c's are to be so chosen that the a's are integral functions. Applying these formulas, we have

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots=\cfrac{x}{1+\cfrac{x^2}{2.3-x^2+\cfrac{2.3x^2}{4.5-x^2+\dots}}}$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots=\cfrac{1}{1+\cfrac{x^2}{1.2-x^2+\cfrac{1.2x^2}{3.4-x^2+\cfrac{3.4x^2}{5.6-x^2+\dots}}}}$$

$$\tan x=\frac{2}{\pi-2x}-\frac{2}{\pi+2x}+\frac{2}{3\pi-2x}-\frac{2}{3\pi+2x}+\frac{2}{5\pi-2x}+\dots^{*}$$

$$=\cfrac{2}{\pi-2x+\cfrac{4x}{(\pi-2x)^2+\cfrac{2\pi}{\pi^4-4x^2+\cfrac{(\pi-2x)(3\pi-2x)^2}{2x(\pi+2x)+\dots}}}}$$

**Encyclopedia Britannica*, Vol. XXIII, p. 572.

$$\cot x = \frac{1}{x} - \frac{1}{\pi - x} + \frac{1}{\pi + x} - \frac{1}{2\pi - x} + \frac{1}{2\pi + x} - \ldots^{*}$$

$$= \cfrac{1}{x + \cfrac{x^2}{\pi - 2x + \cfrac{(\pi - x)^2}{2x + \cfrac{(\pi + x)^2}{(\pi - 2x) + \ldots}}}}$$

$$\sec x = \frac{4\pi}{\pi^2 - 4x^2} - \frac{3.4\pi}{3^2\pi^2 - 4x^2} + \frac{4.5\pi}{5^2\pi^2 - 4x^2} - \frac{4.7\pi}{7^2\pi^2 - 4x^2} + \frac{4.9\pi}{9^2\pi^2 - 4x^2} - \ldots\dagger$$

$$= \cfrac{4\pi}{\pi^2 - 4x^2 + \cfrac{3(\pi^2 - 4x^2)^2}{6\pi^2 + 8x^2 + \cfrac{(9\pi^2 - 4x^2)^2}{30\pi^2 + 8x^2 + \ldots}}}$$

$$\csc x = \frac{1}{x} + \frac{2x}{\pi^2 - x^2} - \frac{2x}{2^2\pi^2 - x^2} + \frac{2x}{3^2\pi^2 - x^2} - \frac{2x}{4^2\pi^2 - x^2} + \ldots$$

$$= a - \frac{ax - 1}{x} + \frac{2x}{\pi^2 - x^2} - \frac{2x}{4\pi^2 - x^2} + \ldots\ddagger$$

$$= \cfrac{a}{1 + \cfrac{ax - 1}{1 + \cfrac{2ax^3}{a\pi^2x - \pi^2 - x^2 - ax^3 + \cfrac{2x(ax - 2)(\pi^2 - x^2)^2}{6\pi^2 + \ldots}}}}$$

Also solved by G. W. Greenwood.

268. Proposed by O. E. GLENN, Ph. D., Philadelphia, Pa.

Express the hyperbolic functions of x in the form of infinite continued fractions.

Solution by J. SCHEFFER, A. M., Professor of Mathematics. Kee Mar College, Hagerstown, Md.

For the conversion of $\tan x$ and $\tanh x$, Legendre has given a very elegant solution.

Putting $f(x) = 1 + \frac{a}{x} + \frac{1}{2}.\frac{a^2}{x(x+1)} + \frac{1}{2.3}.\frac{a^3}{x(x+1)(x+2)} + \ldots$ we have

$$f(x+1) = 1 + \frac{a}{x+1} + \frac{1}{2}.\frac{a^2}{(x+1)(x+2)} + \frac{1}{2.3}.\frac{a^3}{(x+1)(x+2)(x+3)} + \ldots$$

$$\therefore f(x) - f(x+1) = \frac{a}{x(x+1)} + \frac{a^2}{x(x+1)(x+2)} + \frac{1}{2}\frac{a^3}{x(x+1)(x+2)(x+3)} + \ldots$$

** Encyclopedia Britannica*, Vol. XXIII, p. 572.

† Euler's *Institu. Calc. Diff.*, II, 8, No. 223—224.

‡ The quantity a may have any integral value which will avoid negative terms in the continued fraction.